Overview of lecture

• What is a mixed (or split plot) design
• Partitioning the variability
• Pre-analysis checks
• Example ANOVA
• Reporting the Results
• Summary of ANOVA

Mixed or Split Plot Designs

• When there is more than one factor, we can have a mixed (or split plot) design:
  • One or more repeated measures
  • One or more between subjects measures
• The Mixed or Split Plot design combines features of both between groups and within subjects designs:
  • That is each level of factor A contains a different groups of randomly assigned subjects.
  • On the other hand, each level of factor B at any given level of factor A contains the same subjects
  • No such thing as a one factor mixed design.

Partitioning the variance

• Partitioning the variance is done as for a standard ANOVA with small variations
• For the effects of interest
  • A between groups effect is estimated
  • A within subjects effect is estimated
  • An interaction effect is estimated
• For the error terms
  • A between subjects error is used for the between groups effect
  • Within subjects error term used for the within subjects effect
  • The within subjects error term is also used for the interaction effect since this includes a within subject component.

F ratios

• For the main effects and the interaction there are separate F ratios calculated

\[ F_A = \frac{MS_A}{MS_{error}} \]
\[ F_B = \frac{MS_B}{MS_{error}} \]
\[ F_{AB} = \frac{MS_{AB}}{MS_{error}} \]

Complete Example

• Assessing a drug treatment to reduce systolic blood pressure

Pre-analysis checks

• Homogeneity of Variance
  • For a mixed design it is necessary to check in a variety of different ways
    • Overall - Box’s M
    • Within subjects - Mauchly’s W
    • Between groups - Levene’s test
• Normality
  • This can also be tested in a number of ways
  • SPSS conduct’s these tests.
  • Box’s Test of Equality of Covariance Matrices
Homogeneity of variance

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squares</th>
<th>df1</th>
<th>df2</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box M</td>
<td>4,189</td>
<td></td>
<td>1</td>
<td>488</td>
<td>.185</td>
</tr>
<tr>
<td>df2</td>
<td>724,529</td>
<td></td>
<td></td>
<td>.534</td>
<td></td>
</tr>
</tbody>
</table>

Mauchly's W = .858
Approx. Chi-Square = 11.05
Sig. = .034

Levene's Test

<table>
<thead>
<tr>
<th>Time</th>
<th>Sum of Squares</th>
<th>df1</th>
<th>df2</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blood Pressure (1 hr)</td>
<td>.000</td>
<td>1</td>
<td>10</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Blood Pressure (5 hrs)</td>
<td>4.870</td>
<td>1</td>
<td>10</td>
<td>10.06</td>
<td></td>
</tr>
<tr>
<td>Blood Pressure (9 hrs)</td>
<td>0.000</td>
<td>1</td>
<td>10</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Sig. = .000

Summary of omnibus F results

- A significant main effect of treatment
- No further analyses required
- A significant main effect of time
- Post hoc tests required (no a priori prediction)
- A significant interaction between treatment and time
- Simple main effect analysis required

Main effect of time

- Post hoc tests - Tukey

Comparison between means for selected factor(s)

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squares</th>
<th>df1</th>
<th>df2</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment at 1 hour</td>
<td>2352,000</td>
<td>1</td>
<td>10</td>
<td>23.52</td>
<td>.0000</td>
</tr>
<tr>
<td>5 hours</td>
<td>3880,000</td>
<td>1</td>
<td>10</td>
<td>38.80</td>
<td>.0000</td>
</tr>
<tr>
<td>9 hours</td>
<td>75,000</td>
<td>1</td>
<td>10</td>
<td>7.50</td>
<td>.1094</td>
</tr>
<tr>
<td>Error Term</td>
<td>1240,000</td>
<td>30</td>
<td>41,600</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Time at Drug

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squares</th>
<th>df1</th>
<th>df2</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drug</td>
<td>3484,000</td>
<td>2</td>
<td>174,000</td>
<td>34.84</td>
<td>.0000</td>
</tr>
<tr>
<td>Placebo</td>
<td>268,000</td>
<td>2</td>
<td>134,000</td>
<td>26.80</td>
<td>.0000</td>
</tr>
<tr>
<td>Error Term</td>
<td>680,000</td>
<td>20</td>
<td>34,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Simple main effects of Drug x Time interaction

- A significant main effect of Drug
- A significant main effect of Time
- A significant interaction between Drug and Time

Tukey test

Comparison between levels of Time

<table>
<thead>
<tr>
<th></th>
<th>Sum of Squares</th>
<th>df1</th>
<th>df2</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 hour vs 5 hours</td>
<td>2.97</td>
<td>10</td>
<td>10</td>
<td>3.97</td>
<td>.0036</td>
</tr>
<tr>
<td>1 hour vs 9 hours</td>
<td>10.99</td>
<td>8</td>
<td>10</td>
<td>10.99</td>
<td>.0001</td>
</tr>
<tr>
<td>5 hours vs 9 hours</td>
<td>8.02</td>
<td>8</td>
<td>10</td>
<td>8.02</td>
<td>.0001</td>
</tr>
</tbody>
</table>
Summary of simple main effects

- A significant simple main effect of Treatment at 1 hour
- The blood pressure for the drug and placebo groups are significantly different at 1 hour
- A significant simple main effect of Treatment at 5 hours
- The blood pressure for the drug and placebo groups are significantly different at 5 hours
- A non-significant simple main effect of Treatment at 9 hours
- The blood pressure for the drug and placebo groups are not significantly different at 9 hours
- A significant simple main effect of Time at Drug
- Post hoc tests required
- A significant simple main effect of Time at Placebo
- Post hoc tests required

Post hoc analysis of simple main effects

Comparison between levels of Time

\begin{align*}
\text{at level Drug} \\
1 \text{ hour vs 5 hours} & \quad q = 0.42 \\
1 \text{ hour vs 9 hours} & \quad q = 12.40 \quad \text{***} \\
5 \text{ hours vs 9 hours} & \quad q = 12.18 \quad \text{***} \\
\text{at level Placebo} \\
1 \text{ hour vs 5 hours} & \quad q = 3.78 \quad \text{*} \\
1 \text{ hour vs 9 hours} & \quad q = 2.94 \\
5 \text{ hours vs 9 hours} & \quad q = 0.84
\end{align*}

Reporting the results

A two-way (2x3) mixed analysis of variance was conducted on systolic blood pressure. The independent variables included one between groups variable, treatment, with two levels (drug, placebo) and one within subject variable, time, with three levels (1 hour, 5 hours and 9 hours).

Table 1: Means (and standard deviations) of blood pressure for the drug and placebo treatment groups for three times post administration.

<table>
<thead>
<tr>
<th>time</th>
<th>Drug (mmHg)</th>
<th>Treatment (mmHg)</th>
<th>Placebo (mmHg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160 (4.517)</td>
<td>167 (4.500)</td>
<td>162 (5.900)</td>
</tr>
<tr>
<td>5</td>
<td>169 (5.000)</td>
<td>170 (5.800)</td>
<td>169 (6.573)</td>
</tr>
<tr>
<td>9</td>
<td>167 (5.000)</td>
<td>169 (5.800)</td>
<td>162 (5.900)</td>
</tr>
</tbody>
</table>

Simple main effects of Treatment

Simple main effects of Time

Reporting the results

- Report the main effects after the descriptive statistics

  “There was a significant main effect of treatment ($F_{1,10}=43.820$, $\text{MSE}=56.800$, $p<0.001$). Overall the systolic blood pressure of the drug group (142.33) was less than that of the placebo group (165.33).”

  “There was a significant main effect of time ($F_{2,10}=32.324$, $\text{MSE}=34.990$, $p<0.001$). Post hoc Tukey tests (at $p=0.05$) were conducted to examine further the effect of time. The average systolic blood pressure at 9 hours (164.50) after administration of the treatment was significantly greater than at 5 hours (151.00) and at 1 hour (146.00). The latter were not significantly different.”
Reporting the results

"There was a significant interaction between treatment and time \( (F_{2,20} = 22.853, \text{MSE}=34.000, p<0.001; \text{see Table 1}) \). Simple main effects analysis demonstrated that the drug and placebo groups' systolic blood pressure was significantly different at 1 hour \( (F_{1,30} = 56.538, \text{MSE}=41.600, p<0.001) \) and 5 hours \( (F_{1,30} = 93.462, \text{MSE}=41.600, p<0.001) \) but not at 9 hours \( (F_{1,30} = 1.803, \text{MSE}=41.600, p=0.189) \). At both 1 hour and 5 hours the systolic blood pressure was greater for the placebo group than drug group."

"The simple main effect of time was significant for both the drug group \( (F_{2,20} = 51.235, \text{MSE}=34.000, p<0.001) \) and the placebo group \( (F_{2,20} = 3.941, \text{MSE}=34.000, p<0.05) \). Post hoc Tukey tests (at \( p \leq 0.05 \)) were conducted to explore further these effects. For the simple main effect of time for the drug group blood pressure was different between 1 hour and 9 hours and between 5 hours and 9 hours. However, blood pressure was not different between 1 and 5 hours. Blood pressure was lower at 1 hour and 5 hours than at 9 hours for the drug group. For the placebo group, there was one significant difference between 1 hour and 5 hours with blood pressure at 5 hours being significantly greater."

Summary of ANOVA

- ANOVA is a parametric statistical technique for testing the differences between means.
- ANOVA can be used to analyse both single factor and multifactorial designs.
- Anova can be used to analyse differences in between ANOVA, within subjects and mixed designs.

Summary of ANOVA

- A number of assumptions are made by ANOVA that should be tested prior to analysis
- Significant results often require further analysis
- Both planned and unplanned comparisons can be conducted
- Interactions nearly always require further analysis
- A failure to find a significant result may be due to lack of statistical power