functional Magnetic Resonance Imaging – Methods

Denis Schluppeck

Visual Neuroscience Group
University of Nottingham, UK

2/4

Next ~~4~~ 3 lectures

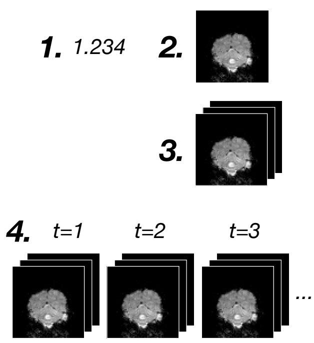
1. Spatial and temporal properties of fMRI
   (+ linearity, convolution)
2. Signal and Noise
   (+ Fourier domain, convolution)
3. Preprocessing of fMRI data
   (+ common software tools)
4. Statistics + experimental design
   (+ linear regression, GLM, multiple comparisons)

Matlab

?

Quick recap: data

1. numbers (=pixel/voxel)
2. a bunch of numbers on a grid (=slice)
3. a collection of slices (=volume)
4. many volumes over time, acquired every TR (=timeseries)



Data: indexing

- if we have a timeseries of volumes (in 3D + 1D = 4D), we need 4 "indices" or coordinates to uniquely identify a voxel (x,y,z,t)
- multi-dimensional arrays
- we can **slice** this data in different ways:

```
>> data(:,:,12,1) % get slice z=12 at t=1
>> data(32,:,:,1) % ??
>> data(1,1,12,:) % get timeseries at [1,1,12]
```

Data: indexing

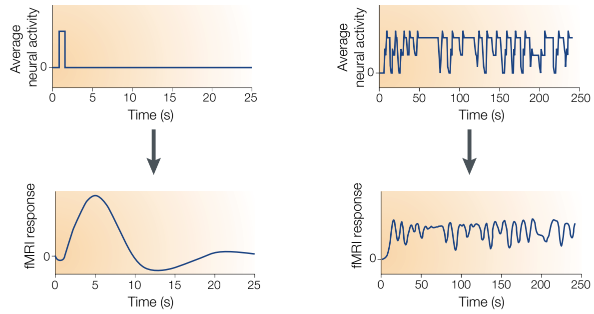
- if we have a timeseries of volumes (in 3D + 1D = 4D), we need 4 "indices" or coordinates to uniquely identify a voxel (x,y,z,t)
- multi-dimensional arrays
- we can **slice** this data in different ways:

```
>> data(:,:,12,1) % get slice z=12 at t=1
>> data(32,:,:,1) % y/z slice at x=32, t=1
>> data(1,1,12,:) % get timeseries at [1,1,12]
```

HRF

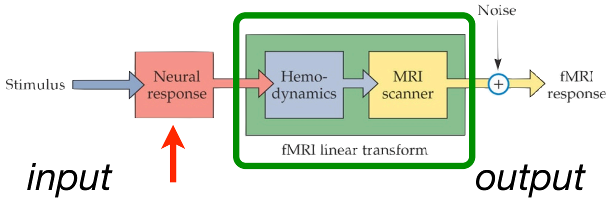
- the shape of the response to a brief impulse (e.g. visual stimulus) is called the haemodynamic response function (HRF)
- for a *linear system*, knowing the impulse response is sufficient to predict the response to an arbitrary input
- Linearity – clarification...
- Fourier domain / convolution
- Signal-to-noise / contrast-to-noise

impulse response *linear prediction*



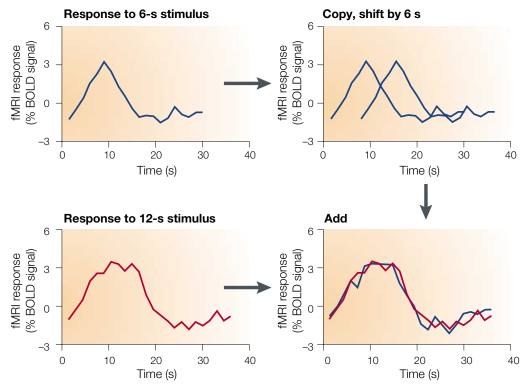
Heeger & Ress, NRN (2002)

fMRI response as a linear system

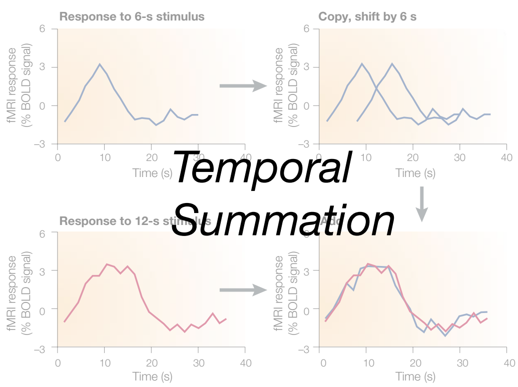


input *output*

Boynton et al (1996)



Heeger & Ress, NRN (2002)



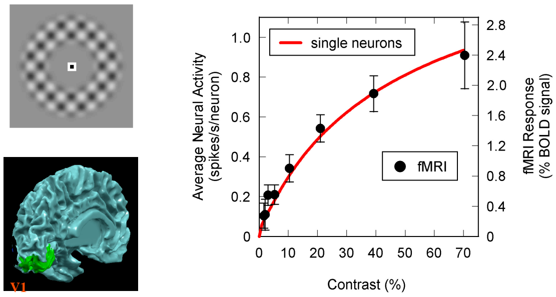
Temporal Summation

Heeger & Ress, NRN (2002)

Neural activity: input to fMRI transform

1. fMRI response is approximately a linear system
2. neural activity is **not** a linear transform of e.g. visual stimulus
   - neuronal firing rates are > 0 (so at least half-rectifying)
   - response to visual contrast saturates (contrast response function)

fMRI response, firing rates



Heeger et al (2000) Nature Neurosci, 3:631+

Linearity does not always hold

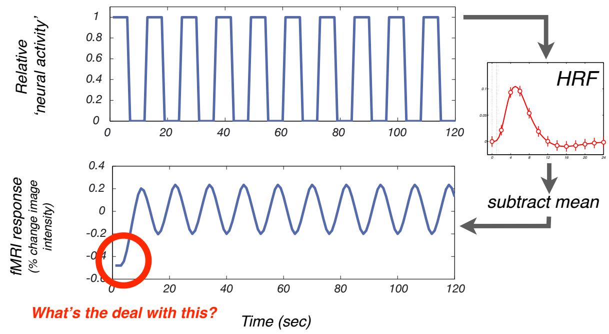
1. very brief events (threshold)
2. “refractory” effects for very closely spaced events

   cf. fMRI adaptation

Simulation
fMRI Response in a block design experiment

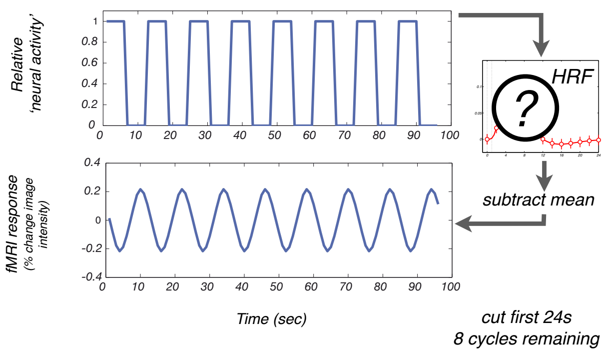
Block alternation

Stimulus alternation frequency = 1/12 Hz; (12s cycle)



Block alternation

Stimulus alternation frequency = 1/12 Hz; (12s cycle)

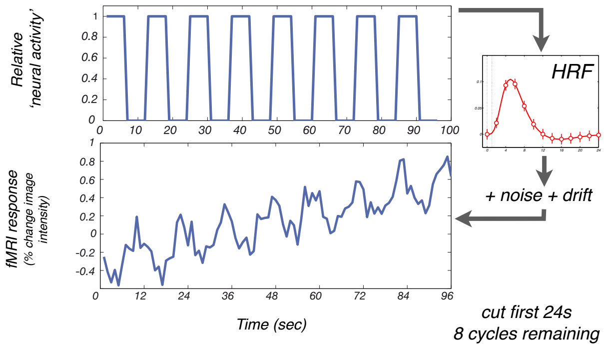


Noise

- measured data is never perfect...
- sources of (unwanted) variability: *heart beat, breathing, movements, ...*
- in fMRI data we usually (**high-frequency) ‘noise’** and **drift**

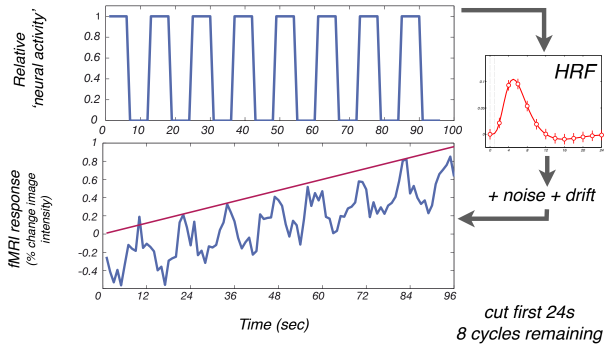
Block alternation / drift

Stimulus alternation frequency = 1/12 Hz; (12s cycle)



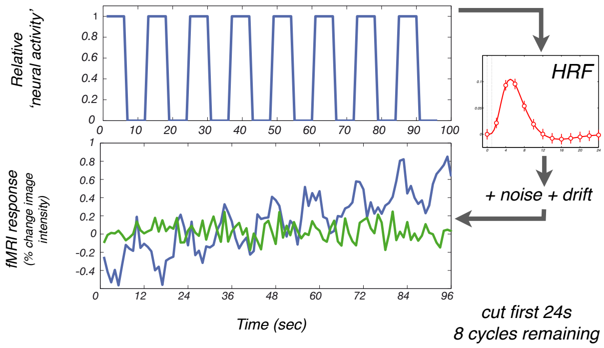
Block alternation / drift

Stimulus alternation frequency = 1/12 Hz; (12s cycle)



Block alternation / noise

Stimulus alternation frequency = 1/12 Hz; (12s cycle)



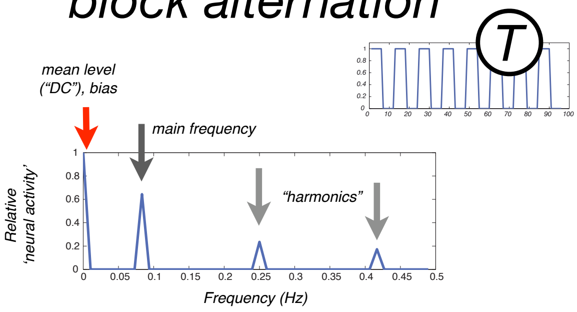
Time / Fourier Domain

Time domain versus $\mathcal{F}$ourier domain

- compare to what you know about image domain ↔ *k*-space
- two different ways of looking at a signal: one in terms of time: ***s, ms***, the other in terms of frequencies: ***Hz (s^{-1}), cycles/scan***
- *Mathtools (Eero Simoncelli, NYU)* ***http://www.cns.nyu.edu/~eero/math-tools/*** *contains additional links to www / books*

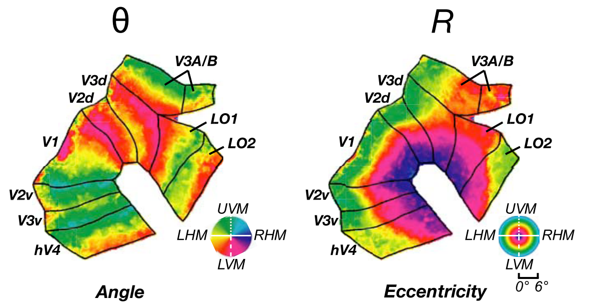
T		$\mathcal{F}$
Time / signal domain	x $\xrightarrow{\text{'Fourier transform'}}$ y	*Frequency domain*
	x $\xleftarrow{\text{'Inverse Fourier transform'}}$ y
data		*complex numbers*
intensity as a function of time		*frequency phase*

Fourier transform of block alternation



Where do the specific numbers come from?
1/12 s = 0.083Hz

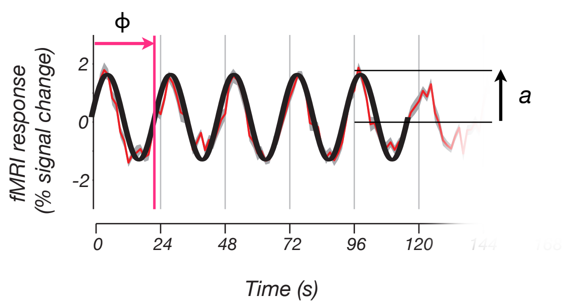
Retinotopic / topographic Mapping
Lecture on "Vision" by Dr Peirce



Larsson & Heeger (2006) J Neurosci

Clever choice of stimulus: map "spatial location" into temporal delay (travelling wave of activity)

amplitude
coherence: [0, 1]
phase: $0 \leq \phi < 2\pi$



Lots of Fourier transforms...

	time domain	*$\mathcal{F}$ourier domain*
Fourier Transform	*continuous, infinite*	*continuous, infinite*
Fourier Series	*continuous, periodic*	*discrete, infinite*
DTFT	*discrete, infinite*	*continuous, periodic*
DFS	*discrete, periodic*	*discrete, periodic*
DFT	***discrete, finite***	***discrete, finite***

FFT Algorithm

- Computes DFT (discrete Fourier Transform) of finite length input
- Very efficient for inputs of lengths $N = 2^n$
- Produces 2 outputs, each of size/length equal to that of the input:
  real part (cosine coefficients)
  imaginary part (sine coefficients)

```
>> fftdemo      % matlab
```

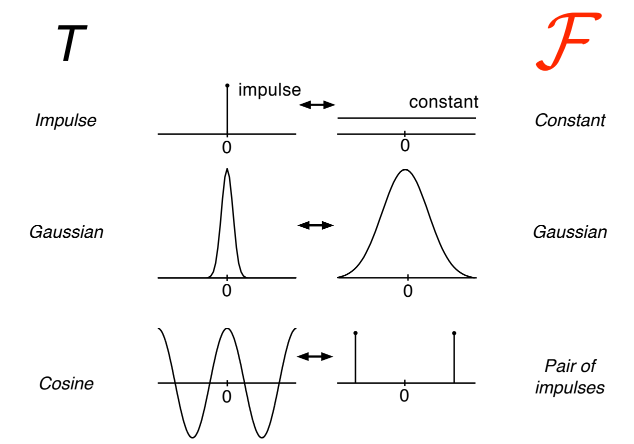
T — *Time / signal domain*

$\mathcal{F}$ — *Frequency domain*

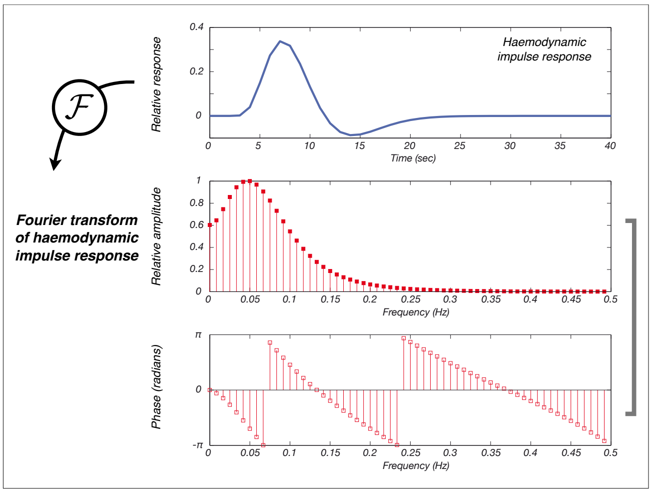
x $\xrightarrow{\text{'Fourier transform'}}$ y
`fft(x)`

x $\xleftarrow{\text{'Inverse Fourier transform'}}$ y
`ifft(y)`

```
help fft, help ifftshift
help ifft, help abs
```



Eero Simoncelli, NYU



Convolution

Convolution

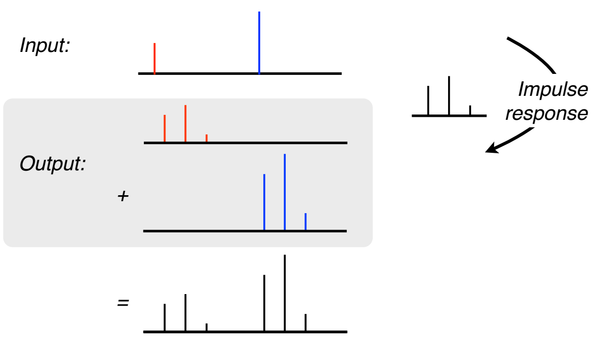
Discrete-time signal: $x[n] = [x_1, x_2, x_3, \ldots]$

A system or transform maps an input signal into an output signal:
$$y[n] = T\{x[n]\}$$

A shift-invariant, linear system can always be expressed as a convolution:
$$y[n] = \sum x[m] \cdot h[n-m]$$

where h[n] is the impulse response.

Convolution as a sum of impulses



Convolution as sequence of weighted sums

past			present				future		
0	0	0	1	0	0	0	0	0	input (impulse)
	1/8	1/4	1/2 →						weights
0	0	0	1/2	1/4	1/8	0	0	0	output (impulse response)
0	0	0	1	1	1	1	1	1	input (step)
			1/8	1/4	1/2 →				weights
0	0	0	1/2	3/4	7/8	7/8	7/8	7/8	output (step response)

☣ Convolution as matrix multiplication

$$\begin{bmatrix} \cdot \\ 5 \\ 2 \\ -3 \\ 4 \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & & & & & \\ & 1 & 2 & 3 & 0 & 0 & 0 & \\ & 0 & 1 & 2 & 3 & 0 & 0 & \\ & 0 & 0 & 1 & 2 & 3 & 0 & \\ & 0 & 0 & 0 & 1 & 2 & 3 & \\ & & & & & \cdot & \cdot & \cdot \\ & & & & & & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ 1 \\ 2 \\ 0 \\ 0 \\ -1 \\ \cdot \end{bmatrix}$$

Linear system ↔ matrix multiplication

Shift-invariant linear system ↔ ‘Toeplitz’ matrix

Matrix multiplication ??

A is a 2 by 2 matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 3.2 \end{bmatrix}$$

x is a vector (2 by 1 matrix)

Matrix multiplication ??

A is a 2 by 2 matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 3.2 \end{bmatrix}$$

x is a vector (2 by 1 matrix)

$$\begin{bmatrix} 1 \times 2.5 & +0 \times 3.2 \\ 0 \times 2.5 & +2 \times 3.2 \end{bmatrix}$$

Matrix multiplication ??

A is a 2 by 2 matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2.5 \\ 3.2 \end{bmatrix}$$

x is a vector (2 by 1 matrix)

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} 2.5 + \begin{bmatrix} 0 \\ 2 \end{bmatrix} 3.2 = \begin{bmatrix} 2.5 \\ 6.4 \end{bmatrix}$$

weighted sum of columns ... $ax_1 + bx_2$ ... should ring a bell!

☣ Convolution as matrix multiplication

$$\begin{bmatrix} \cdot \\ 5 \\ 2 \\ -3 \\ 4 \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & & & & & \\ & 1 & 2 & 3 & 0 & 0 & 0 & \\ & 0 & 1 & 2 & 3 & 0 & 0 & \\ & 0 & 0 & 1 & 2 & 3 & 0 & \\ & 0 & 0 & 0 & 1 & 2 & 3 & \\ & & & & & \cdot & \cdot & \cdot \\ & & & & & & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ 1 \\ 2 \\ 0 \\ 0 \\ -1 \\ \cdot \end{bmatrix}$$

Columns contain shifted copies of the impulse response.

Linear system ↔ matrix multiplication

Shift-invariant linear system ↔ ‘Toeplitz’ matrix

☣ Convolution as matrix multiplication

$$\begin{bmatrix} \cdot \\ 5 \\ 2 \\ -3 \\ 4 \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & & & & & \\ & 1 & 2 & 3 & 0 & 0 & 0 & \\ & 0 & 1 & 2 & 3 & 0 & 0 & \\ & 0 & 0 & 1 & 2 & 3 & 0 & \\ & 0 & 0 & 0 & 1 & 2 & 3 & \\ & & & & & \cdot & \cdot & \cdot \\ & & & & & & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ 1 \\ 2 \\ 0 \\ 0 \\ -1 \\ \cdot \end{bmatrix}$$

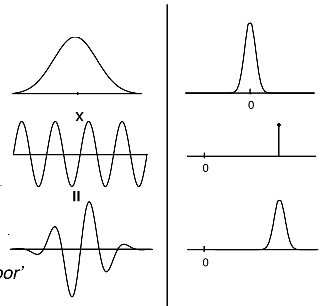
Rows contain time-reversed copies of impulse response.

Linear system ↔ matrix multiplication

Shift-invariant linear system ↔ ‘Toeplitz’ matrix

Convolution Theorem 1

Multiplication in the time domain



Convolution in the frequency domain

easier to calculate here

easier to understand here

Eero Simoncelli, NYU

Convolution Theorem 2

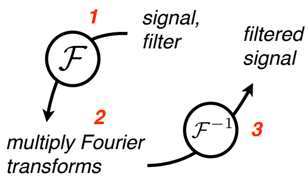
Multiplication in the time domain ⟷ **Convolution** in the frequency domain

Convolution in the time domain ⟷ **Multiplication** in the frequency domain

But why bother with this seemingly complicated business of transforming?

For large data sets

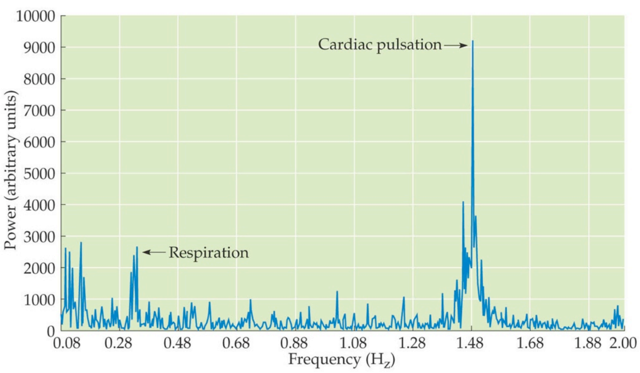
- Convolution is an computationally expensive operation
- FFT / IFFT is very efficient
- Point-by-point multiplication is cheap

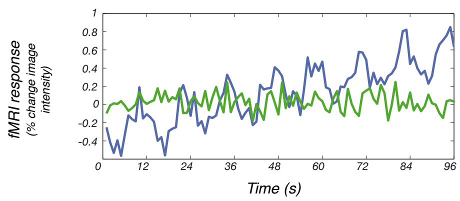


In some cases...

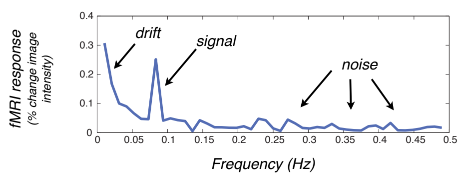
- ... it's easier to see periodic events, e.g. artefacts due to cardiac cycle / breathing in the frequency domain

Lance Armstrong?



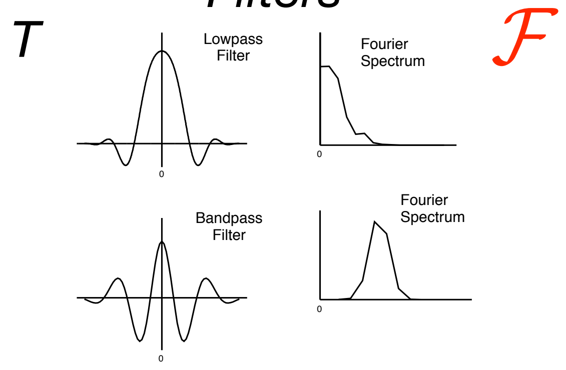


T



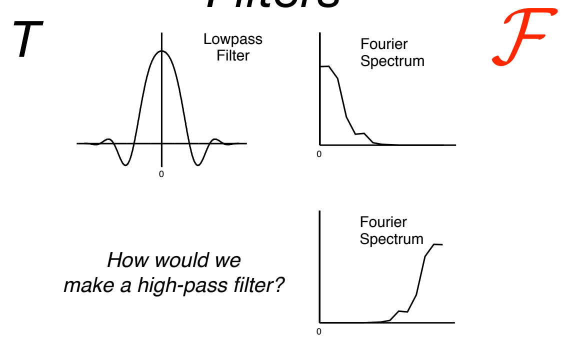
$\mathcal{F}$

Filters



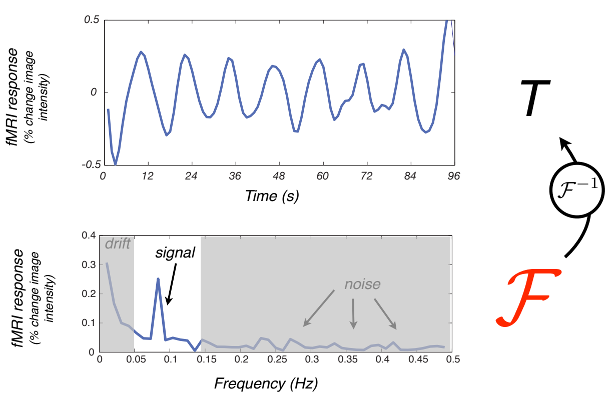
Eero Simoncelli, NYU

Filters



How would we make a high-pass filter?

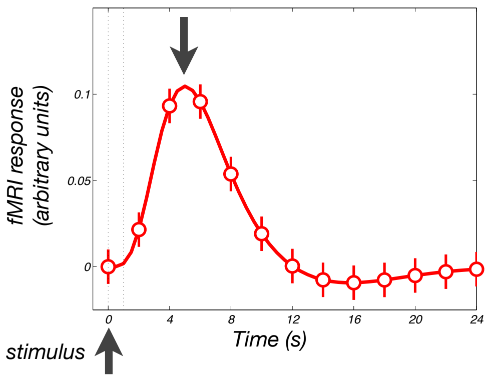
Eero Simoncelli, NYU



☣ Linear Algebra / FFT

- Eero Simoncelli, NYU
  http://www.cns.nyu.edu/~eero/math-tools/
  contains additional links to www / books
- *MIT OpenCourseWare (video lectures)*
  Mathematics, Gilbert Strang, 18.06 course
- *Linear Algebra and Its Applications*, Gilbert Strang, book

☣ HRF



☣ Glover, 1999

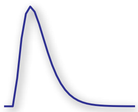
$$H(t) = \left(\frac{t}{d_1}\right)^{a_1} \exp\left(\frac{-(t-d_1)}{b_1}\right) - \left(\frac{t}{d_2}\right)^{a_2} \exp\left(\frac{-(t-d_2)}{b_2}\right)$$

default params [a_1, a_2, b_1, b_2, c] = [6 12 0.9 0.9 0.35]

Glover. Deconvolution of impulse response in event-related BOLD fMRI. *Neuroimage* (1999) vol. 9 (4) pp. 416-29

☣Plot a simple version in Matlab?

$$H(t) = \left(\frac{t}{\tau}\right)^2 \cdot \frac{\exp(-t/\tau)}{2\tau}$$



```
tau = 2; % time constant
delta = 2; % time shift
t = [0:1:30]; % vector of time points
tshift = max(t-delta,0); % shifted, but not < 0
HIRF = (tshift/tau).^2 .* exp(-tshift/tau) ...
/ (2*tau); % function
figure(1), plot(HIRF, ‘r’); % plot it
```

Quantifying Signal / Noise

Signal-to-noise ratio (SNR)

raw SNR: used by physicists + engineers to quantify image quality

/

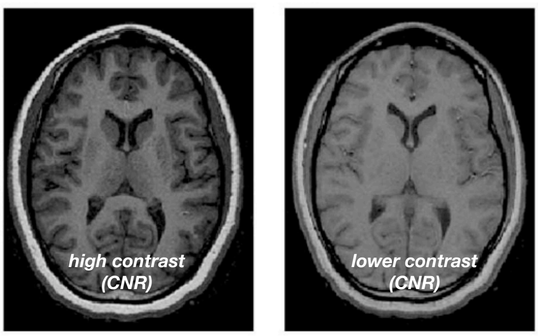
Intensity in sample (e.g. brain) / *Noise outside sample (e.g. outside head)*

2000* / *100

Contrast-to-noise ratio (CNR)

CNR: e.g. how good is T_1 contrast between white matter (WM) and gray matter (GM) – take two small regions of interest

	mean GM		***mean WM***	***noise (σ)***	***cnr***
image 1	*150*	*100*	*250*	*100*	*x*
image 2	*60*	*10*	*70*	*5*	*?*

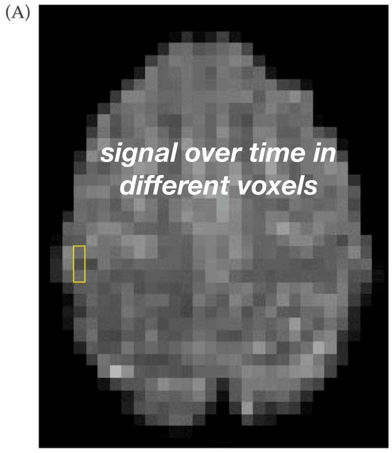


functional signal-to-noise ratio

functional SNR: (sometimes called functional CNR)

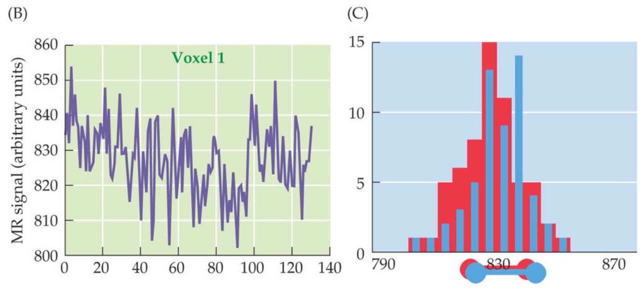
signal: difference between two states of the brain caused by experiment

noise: variability in those states over time...

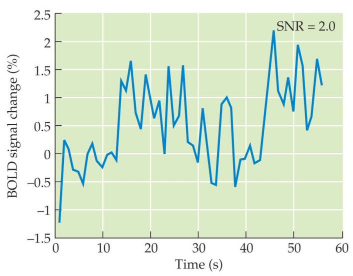


FUNCTIONAL MAGNETIC RESONANCE IMAGING, Figure 9.2 (Part 1) © 2004 Sinauer Associates, Inc.

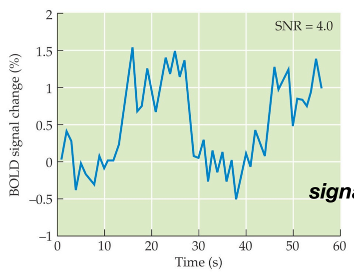
low functional SNR



how to increase functional SNR?

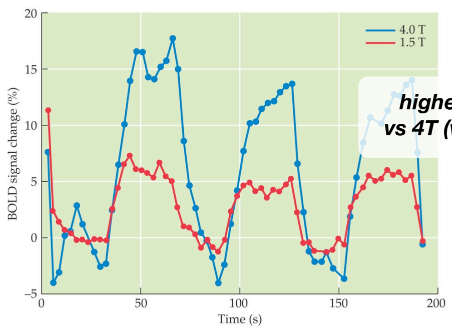


how to increase functional SNR?



signal averaging

how to increase functional SNR?



higher field: 1.5T vs 4T (visual cortex)

Summary

- recap: linear systems
- **Matlab**
- simulated block design data
- drift + (high-frequency) noise
- Fourier domain, convolution
- raw SNR, CNR, functional SNR